The College Board

Advanced Placement Examination

CALCULUS AB

SECTION II

19

Time—1 hour and 30 minutes

Number of problems—7

Per cent of total grade—50

SHOW ALL YOUR WORK. INDICATE CLEARLY THE METHODS YOU USE BECAUSE YOU WILL BE GRADED ON THE CORRECTNESS OF YOUR METHODS AS WELL AS ON THE ACCURACY OF YOUR FINAL ANSWERS.

Notes: (1) In this examination, ln x denotes the natural logarithm of x (logarithm to the base e). (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which f(x) is a real number.

1. Let f be the function defined by $f(x) = x^4 - 3x^2 + 2$.

   (a) Find the zeros of f.

   (b) Write an equation of the line tangent to the graph of f at the point where $x = 1$.

   (c) Find the x-coordinate of each point at which the line tangent to the graph of f is parallel to the line $y = -2x + 4$.

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2. Let R be the region in the first quadrant enclosed by the graphs of $y = 4 - x^2$, $y = 3x$, and the Y-axis.

   (a) Find the area of region R.

   (b) Find the volume of the solid formed by revolving the region R about the X-axis.

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3. Let f be the function defined by $f(x) = 12x^{\frac{2}{3}} - 4x$.

   (a) Find the intervals on which f is increasing.

   (b) Find the x- and y-coordinates of all relative maximum points.

   (c) Find the x- and y-coordinates of all relative minimum points.

   (d) Find the intervals on which f is concave downward.

   (e) Using the information found in parts (a), (b), (c), and (d), sketch the graph of f on the axes provided.

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4. Let f be the function defined by $f(x) = 5^{\sqrt{2x^2-1}}$.

   (a) Is f an even or odd function? Justify your answer.

   (b) Find the domain of f.

   (c) Find the range of f.

   (d) Find $f'(x)$.

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5. Let f be a function defined by

$$f(x) = \begin{cases} 2x + 1, & \text{for } x \leq 2, \\ \frac{1}{2}x^2 + k, & \text{for } x > 2. \end{cases}$$

   (a) For what value of k will f be continuous at $x = 2$? Justify your answer.

   (b) Using the value of k found in part (a), determine whether f is differentiable at $x = 2$. Use the definition of the derivative to justify your answer.

   (c) Let $k = 4$. Determine whether f is differentiable at $x = 2$. Justify your answer.

6. (a) A solid is constructed so that it has a circular base of radius r centimeters and every plane section perpendicular to a certain diameter of the base is a square, with a side of the square being a chord of the circle. Find the volume of the solid.

   (b) If the solid described in part (a) expands so that the radius of the base increases at a constant rate of $\frac{1}{2}$ centimeters per minute, how fast is the volume changing when the radius is 4 centimeters?

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7. Let f be a differentiable function defined for all $x > 0$ such that

   (i) $f(1) = 0$,

   (ii) $f'(1) = 1$, and

   (iii) $\frac{d}{dx}[f(2x)] = f'(x)$, for all $x > 0$.

   (a) Find $f'(2)$.

   (b) Suppose f' is differentiable. Prove that there is a number c, $2 < c < 4$, such that $f''(c) = -\frac{1}{8}$.

   (c) Prove that $f(2x) = f(2) + f(x)$ for all $x > 0$.

END OF EXAMINATION